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# Road and Structure Protection Through Weight Control

## Equivalent Base Length Concept

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# Road and Structure Protection Through Weight Control

## Equivalent Base Length Concept

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## 1/ INTRODUCTION

The design of most highway bridges is predominantly governed by longitudinal moments and shears. The live load portions of these responses are caused by heavy commercial vehicles and are governed by the weights and spacings of their axles. The task of quantifying the heavy vehicles with respect to the load effects they induce in bridges is made difficult by the large number of spans, and by the very large number of axle weight and spacing combinations encountered in practice.

With the help of the concept described here, it is possible to reduce a number of discrete loads into an equivalent uniformly distributed load, which gives nearly the same maximum moments and shears in any beam as the set of discrete loads. This concept has been the basis of the Ontario vehicle weight regulations and is also useful in developing bridge design loads.

## 2/ EQUIVALENT BASE LENGTH, $B_m$

Csagoly and Dorton (1978) have shown that N discrete loads along a beam can be replaced by a uniformly distributed load of total weight, W, and a length, $B_m$, so that the moment envelope along the beam due to this load is very nearly the same as that caused by the discrete loads. The total weight, W, of the uniformly distributed load is equal to the sum of discrete loads, and $B_m$, which is called the **equivalent base length**, is obtained as follows:

$$B_m = \frac{4}{W} \sum_{i=1}^{N} \left| P_i x_i \right| - \frac{2(N-1)}{b\ NW^2} \left[ \sum_{i=1}^{N} (P_i x_i) \right]^2 \qquad (1)$$

where the notation is as shown in Figure 1. The load closest to the centre of gravity of the group of loads is taken as the reference load and distances of other loads, $x_i$, are measured from this load.

It is noted that Equation 1 is independent of the span length of the beam; it only gives approximate values of $B_m$, which as shown later are accurate enough for all practical purposes. Equation 1 is developed from

the following "exact" expression for $B_m$ which incorporates the span length (Jung and Witecki, 1971).

$$B_m = \frac{4}{W} \sum_{i=1}^{N} \left| P_i \chi_i \right| - \frac{2}{LW^2} \left\{ \sum_{i=1}^{N} (P_i \chi_i) \right\}^2 \qquad (2)$$

**2.1/ Accuracy of Equation 1**

The degree of error incurred by Equation 1 is taken as $\Delta$. It is defined as follows:

$$\Delta = \left\{ (M_B/M) - 1 \right\} \times 100 \qquad (3)$$

where $M_B$ is the beam moment caused by the uniformly distributed load of length $B_m$ obtained by Equation 1, and M is the corresponding moment due to the discrete loads. Values of $\Delta$ are plotted in Figure 2a against span length for moments in simply supported beams due to a truck with five axles. It can be seen in this illustrative example that the degree of error is within +1% and -8% for all reference points considered. Values of $\Delta$ are large only where the magnitude of moment is small, and hence, $\Delta$ is relatively insignificant.

Although Equation 1 was developed for moments in simply supported beams, it is also valid for shears and for continuous beams. In Figure 2b, values of $\Delta$ are plotted against the span length of a two-span continuous beam for moments at different points due to a truck with five axles. It will be noted that the values of $\Delta$ are somewhat larger than those of their counterparts in the simply supported beam, but are still small, being within +1% and -10%. In both beams, $\Delta$ reduces with the increase in span lengths.

The actual envelope of moments in a simply supported beam of 10.67 m span due to a five-axle truck, is compared in Figure 3 with that of moments due to the uniformly distributed load of length $B_m$ obtained by Equation 1. The closeness of the two envelopes is striking. The Figure also shows the variation of $\Delta$ along the span. It can be seen that the value of $\Delta$ is very small in the middle half of the bridge, where moments are

usually of concern; they are within ± 5%. Near the supports where the magnitudes of moments are small, Δ becomes as high as about -13%.

Csagoly and Dorton (1973) have shown that the value Δ increases as the number of discrete loads is reduced. However, even for a set of three loads, Δ is found to be within +3% and -11% for simply supported beams, and +3.5% and -13% for two-span continuous beams. It is noted that the above ranges are quoted for moments near end supports. For moments in the middle regions of beams, these ranges are much narrower.

It is concluded that a set of discrete loads can be realistically transformed as a uniformly distributed load without taking account of the beam span.

## 2.2/ W - $B_m$ SPACE

The elimination of the span length from consideration simplifies the job of comparing two sets of discrete loads with regard to their load effects, namely moments and shears, in beams of different spans.

For example, we compare two sets of loads, identified as Vehicle A and B in Figure 4, using the maximum moments they induced in any beam as the basis. If the two vehicles have the same total weight, W, then clearly the vehicle with shorter equivalent base length, $B_m$, will induce higher maximum moments. Alternatively, if the two vehicles have the same $B_m$, then the vehicle with larger W will induce higher maximum moments.

The total longitudinal moment, or shear, across the cross-section of a right bridge due to a single vehicle is used as a basis for comparison and is hereafter referred to simply as the moment, or shear. Thus, the bridge is reduced to a beam and each axle load as a discrete load.

A given vehicle has one value each of W and $B_m$. Therefore, on a two-dimensional surface, defined by W and $B_m$ as the orthogonal axes, one vehicle is represented by a single point. Obviously, the W-$B_m$ space, as this surface is called, can accommodate any number of points. Truck survey data involving a very large number of vehicles can now be condensed on a single sheet of paper. Schematically, such a diagram would

be as shown in Figure 5. An envelope of the type shown in this Figure, or a curve parallel to it, can be used as the basis for vehicle weight control. Such a curve of permissible vehicles can be established after superimposing an envelope of bridge live load capacity of existing bridges and making allowance for a compliance factor, as illustrated in Figure 5. A point outside this curve would indicate a vehicle which would produce higher moments and shears in bridges than vehicles of permissible weights.

A design vehicle may be required to provide an upper bound of maximum moments and shears caused by all existing vehicles. In this case, the $W\text{-}B_m$ points corresponding to the various combinations of the axles of the design vehicle should ideally lie on the envelope curve, as shown in Figure 5. Thus, this envelope could be used as the basis for developing the design vehicle.

Agarwal (1978) has used the $W\text{-}B_m$ space as a convenient tool for comparison of the various vehicle weight regulations across Canada.

## 3/ CONCLUSIONS

It has been demonstrated that a set of point loads representing axle loads of a vehicle can be realistically transformed as a uniformly distributed load of equivalent base length which is independent of the span length of beams. Such an equivalent load produces nearly the same envelope of moments and shears in beams of all spans, as the set of point loads.

The equivalent base length can be used as a tool for comparing truck survey data as they affect moments and shears in beams.

## 4/ REFERENCES

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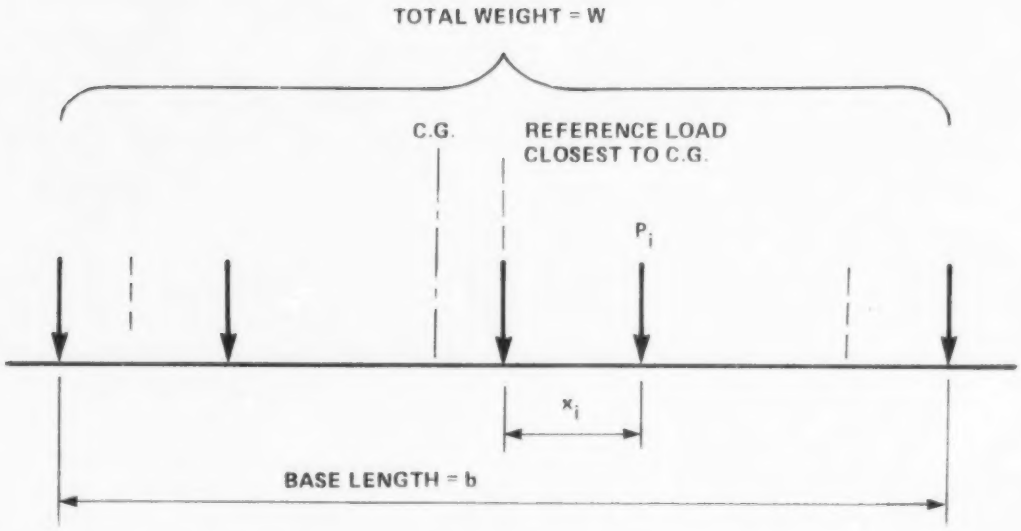


Figure 1/ Notation for Point Loads and Their Spacings

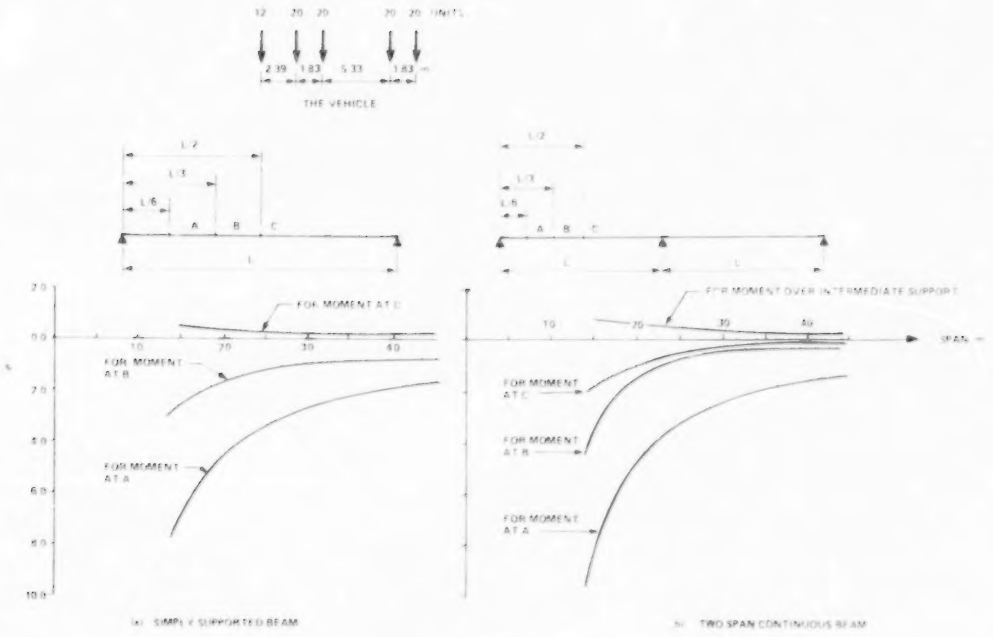


**Figure 2/ Errors in Beam Moments -- Illustrative Example**

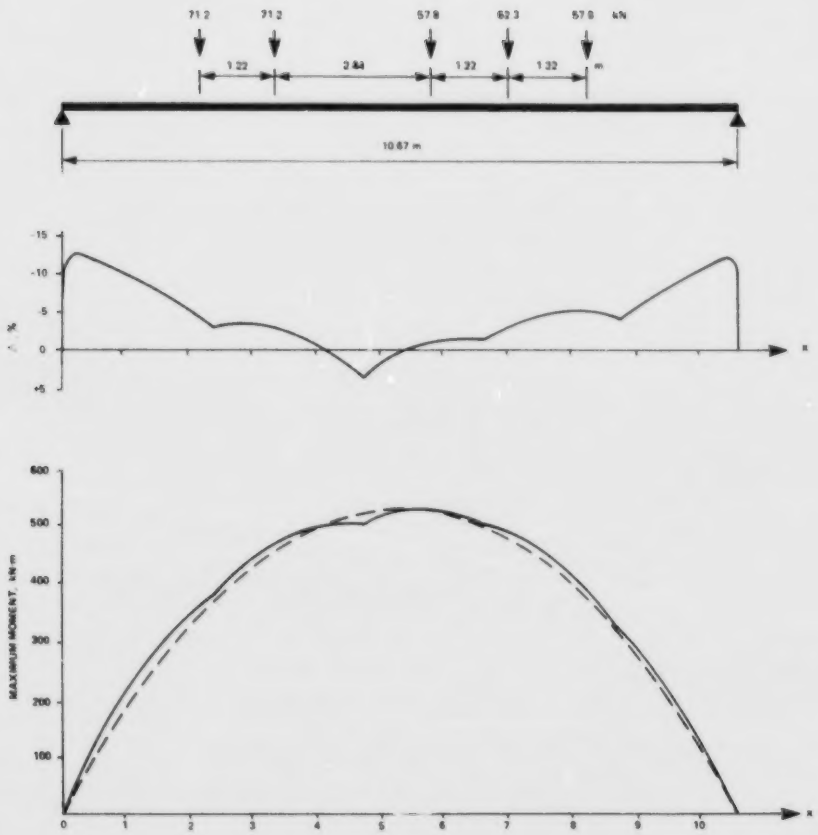


**Figure 3/ Moment Envelopes in a Beam -- Illustrative Example**

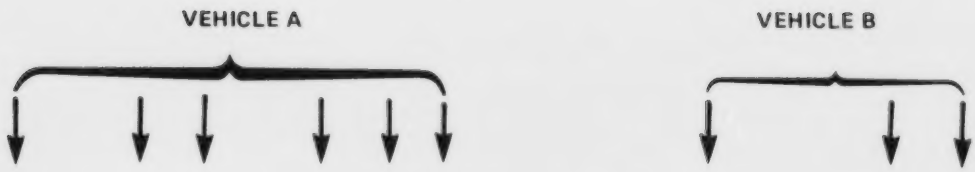


Figure 4/ Two Sets of Loads

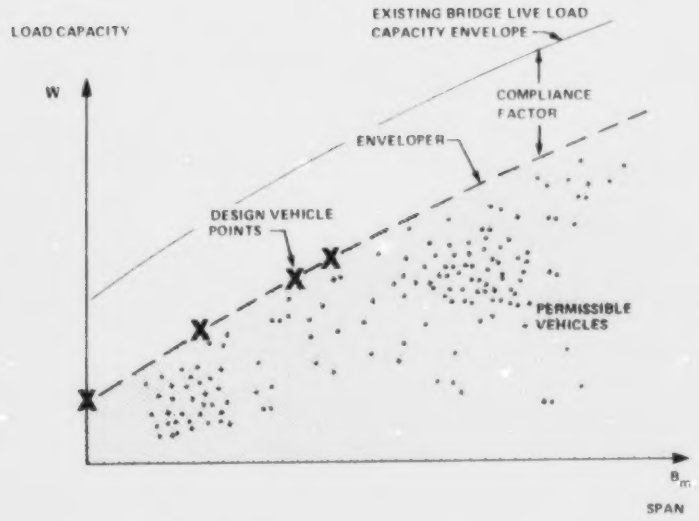


**Figure 5/ Vehicle Data on a W-$B_m$ Space**

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